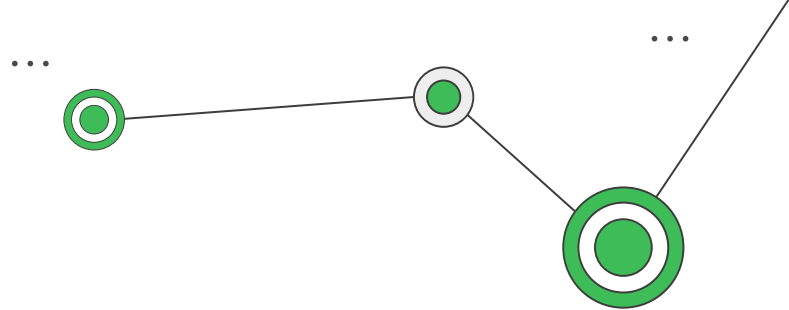
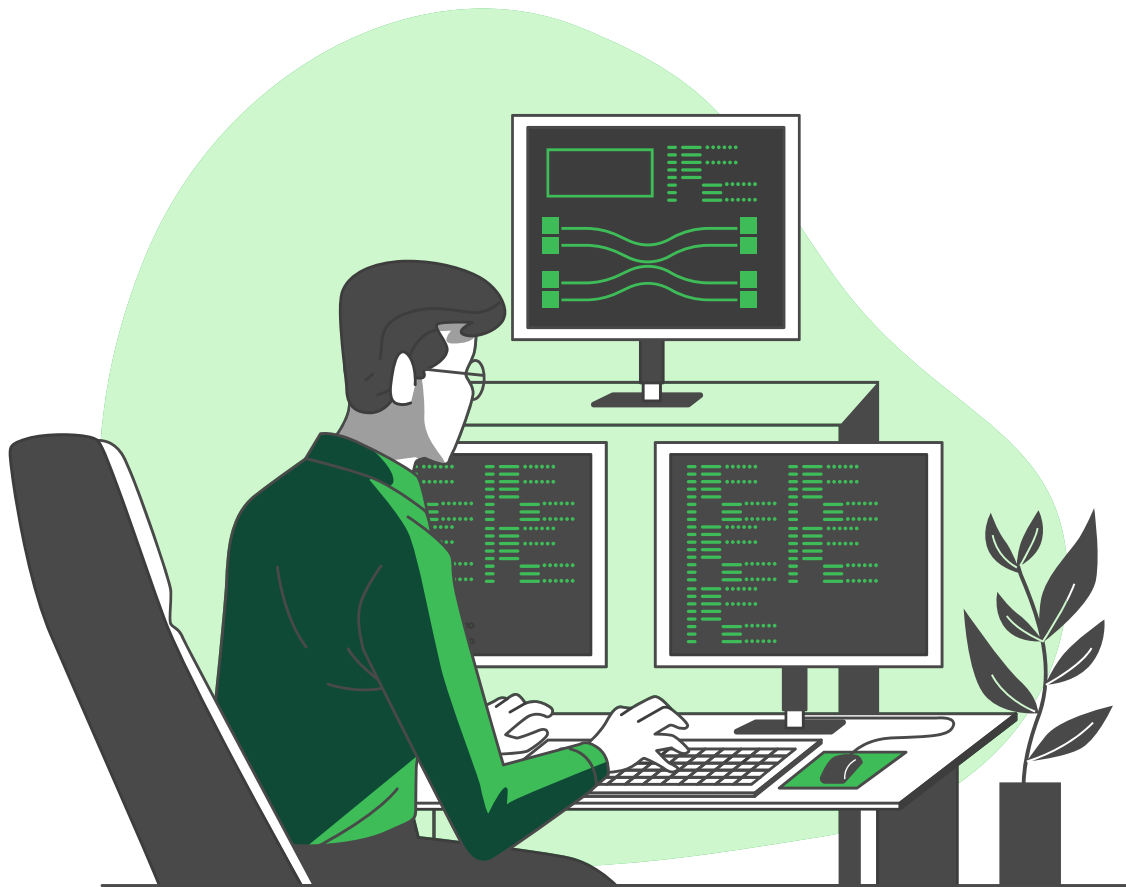
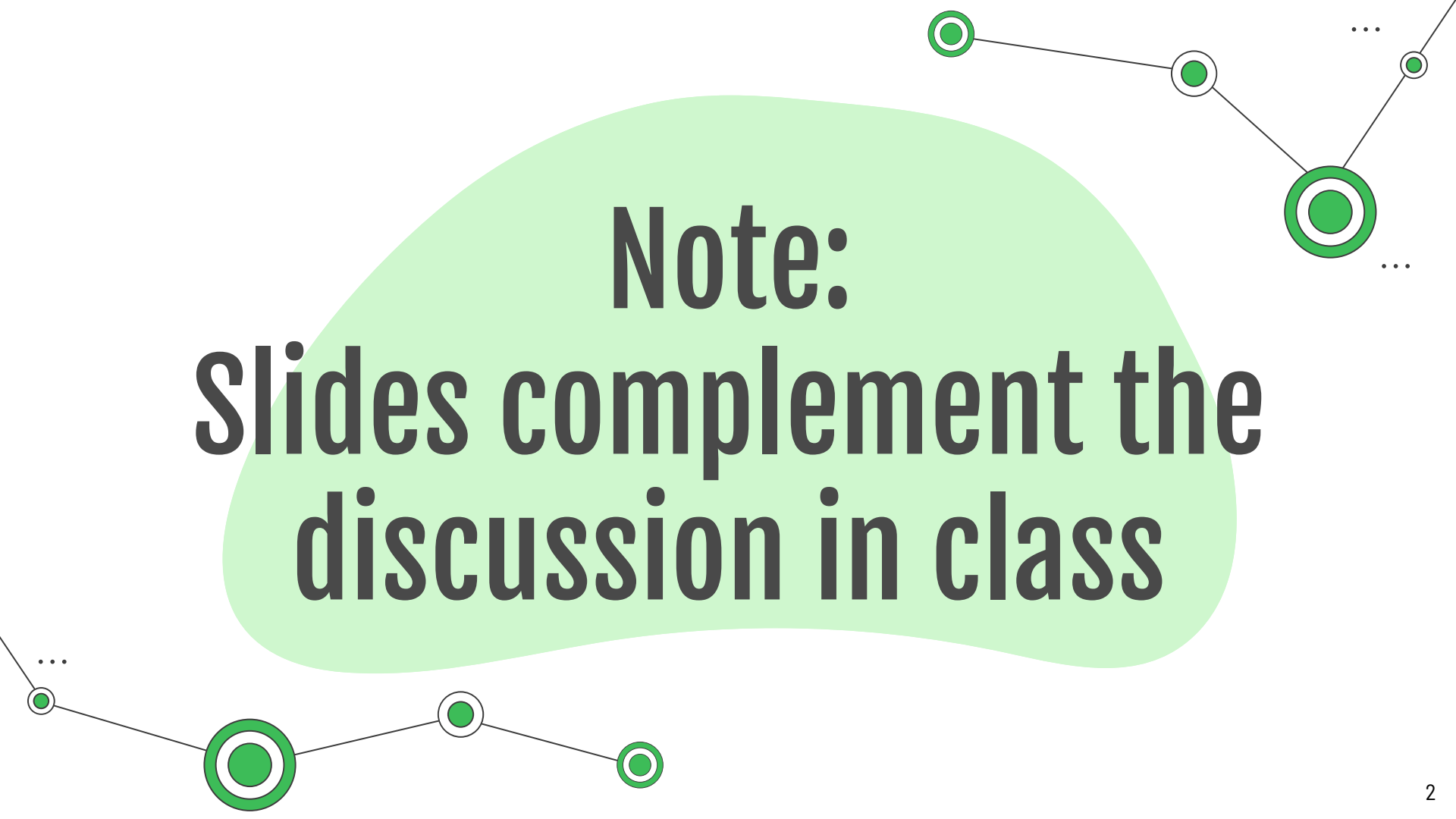




Minimum Spanning Tree

CS 251 - Data Structures
and Algorithms

A decorative network diagram consisting of several green circular nodes connected by thin black lines. Some nodes are single green circles, while others are double green circles. The nodes are arranged in a non-linear fashion, with some at the top right, some at the bottom left, and one in the center. Ellipses (...) are placed near some of the nodes, suggesting a larger network. The central text is overlaid on a light green, irregularly shaped background.

Note:
**Slides complement the
discussion in class**

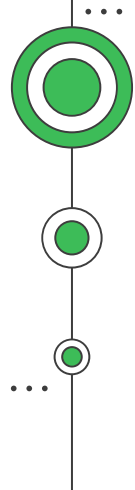


Terminology

Definition and properties

Table of Contents

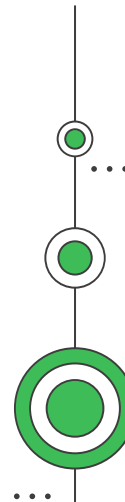




01

Terminology

Definition and properties

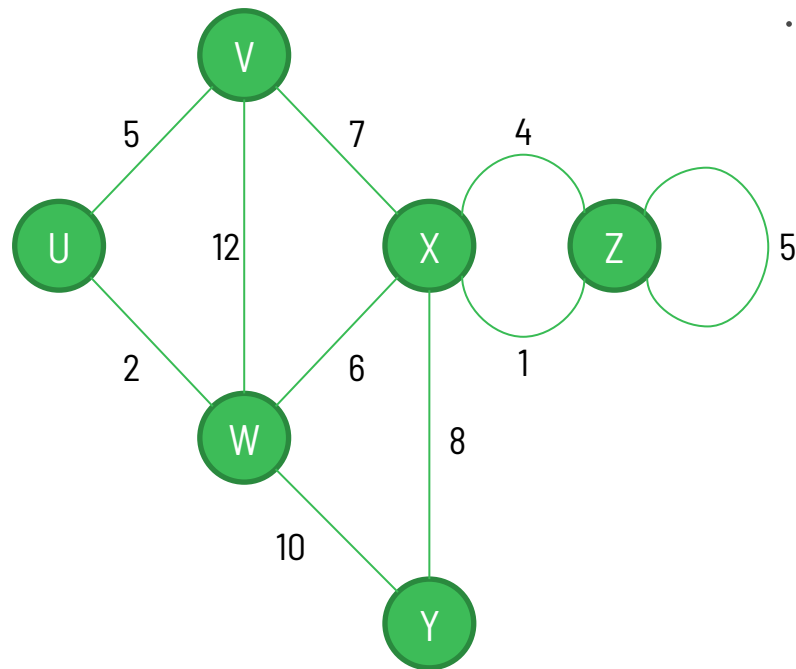


Minimum Spanning Tree

The Minimum Spanning Tree (MST) of a graph G is:

- A spanning subgraph of G (contains all vertices from G)
- A tree (no cycles)
- Minimal weight-wise (sum of edge weights in minimum)

Caution: an MST does not guarantee the shortest path between two vertices.

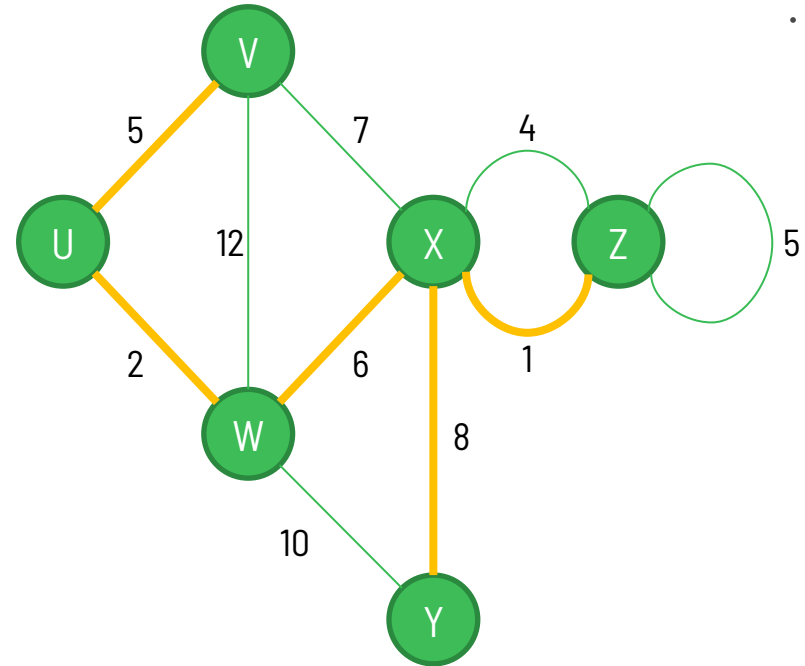


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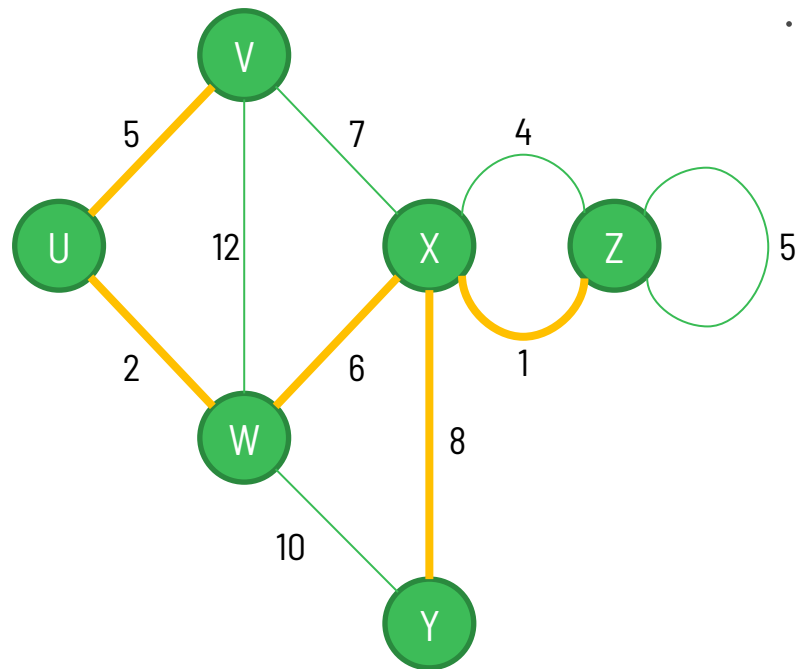
Cycle Property

Let T be a minimum spanning tree of a weighted graph G . Let e be an edge of G that is not in T . Let C be the cycle formed by adding e to T .

Claim:

For every edge $f \in C$, $\text{weight}(f) \leq \text{weight}(e)$

Proof by contradiction: Given the definitions above, assume that for some edge $f \in C$, $\text{weight}(f) > \text{weight}(e)$. Then, replacing f with e will produce a spanning tree T' such that its total weight is smaller than the weight of T . But that contradicts the definition of T as a MST of G .

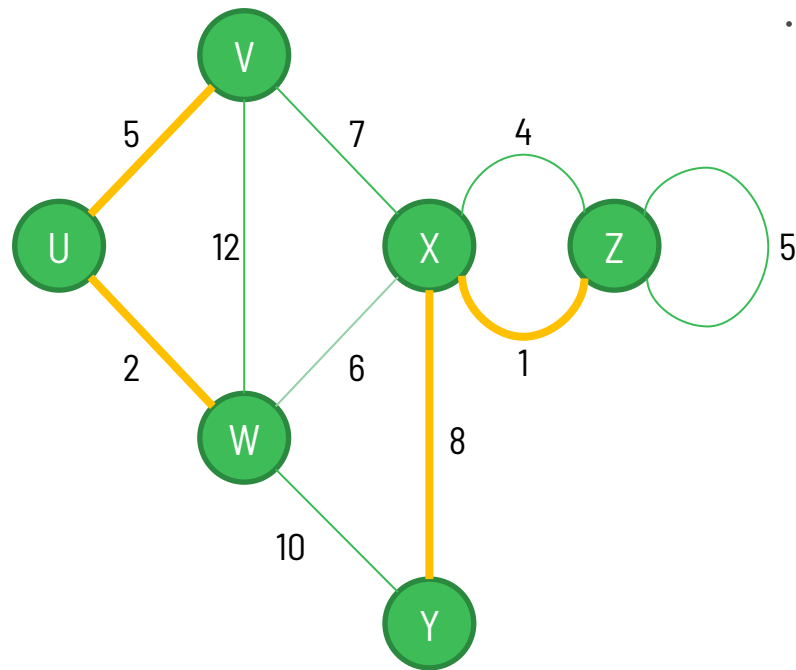


Partition (Cut) Property

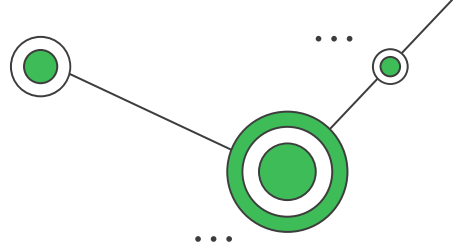
Consider a partition of a graph G into subsets U and V . Let e be an edge of minimum weight across the partition (i.e., e has an endpoint in U and the other in V).

Claim: There is a minimum spanning tree of G that includes edge e .

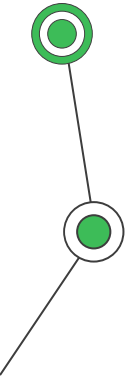
Proof: Let T be an MST of G and let the definitions above be true. If T does not contain e , consider a cycle C formed by e with T and let f be an edge of C across the partition. By the cycle property, $\text{weight}(f) \leq \text{weight}(e)$. Thus, $\text{weight}(f) = \text{weight}(e)$, and we obtain another MST by replacing f with e .



Thinking about MSTs



- Given an undirected, weighted graph G , let T be an MST of G . Is T unique?
- Does an MST also give the shortest path between a pair of vertices?
- Does every weighted undirected graph have an MST?
- Can you find an MST of a weighted undirected graph if there are negative weight edges?
- Can you find an MST in a directed graph?
- How would we find an MST of a weighted undirected graph?



Minimum Viam Connectens

Do you have any questions?

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