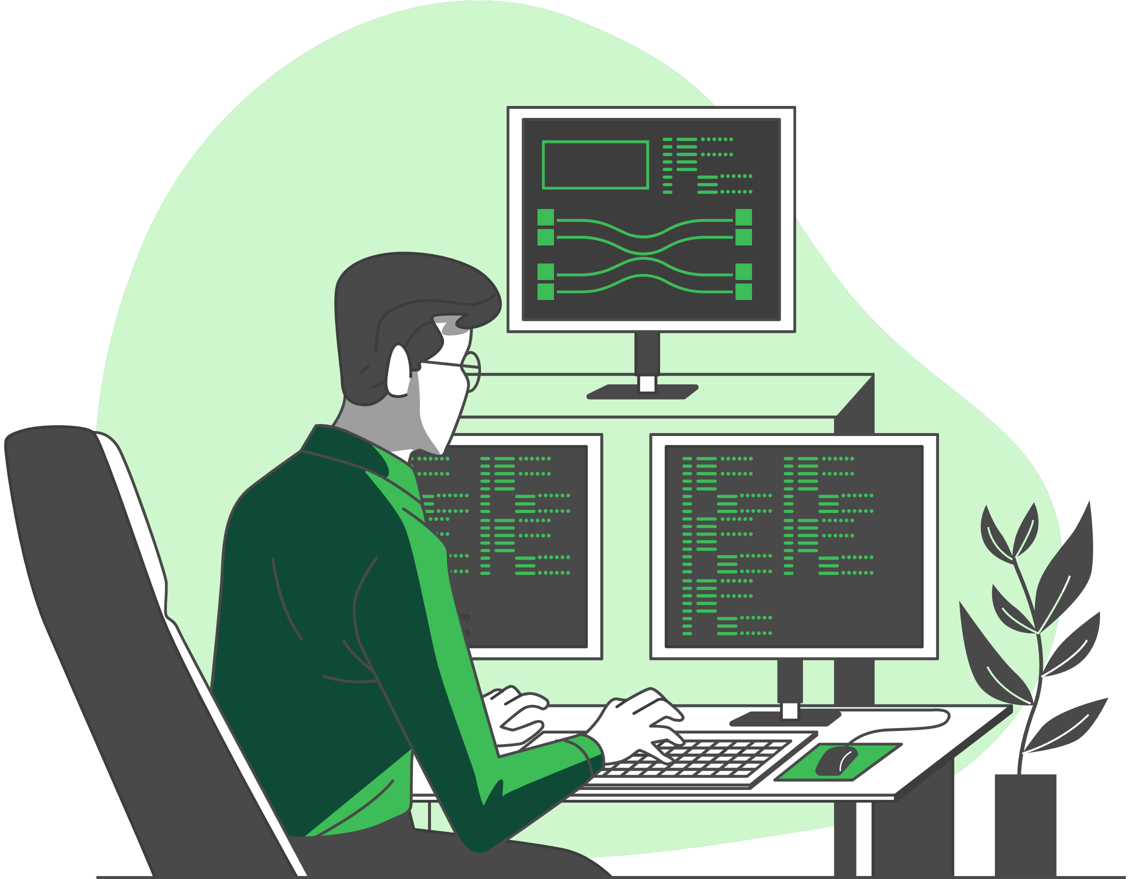

Minimum Spanning Tree

CS 251 - Data Structures and Algorithms

Note:
Slides complement the discussion in class

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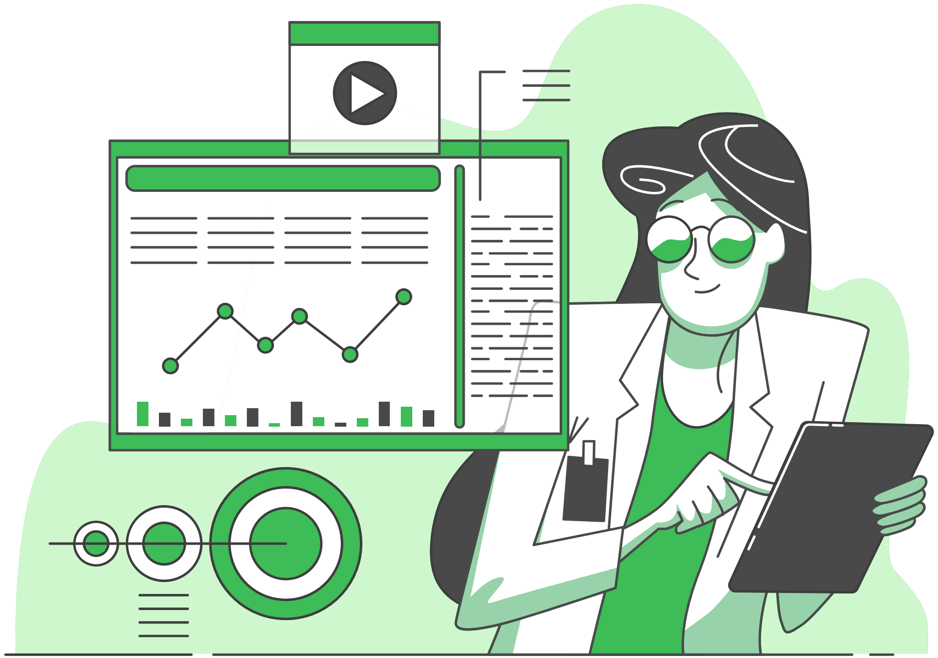


01

Terminology

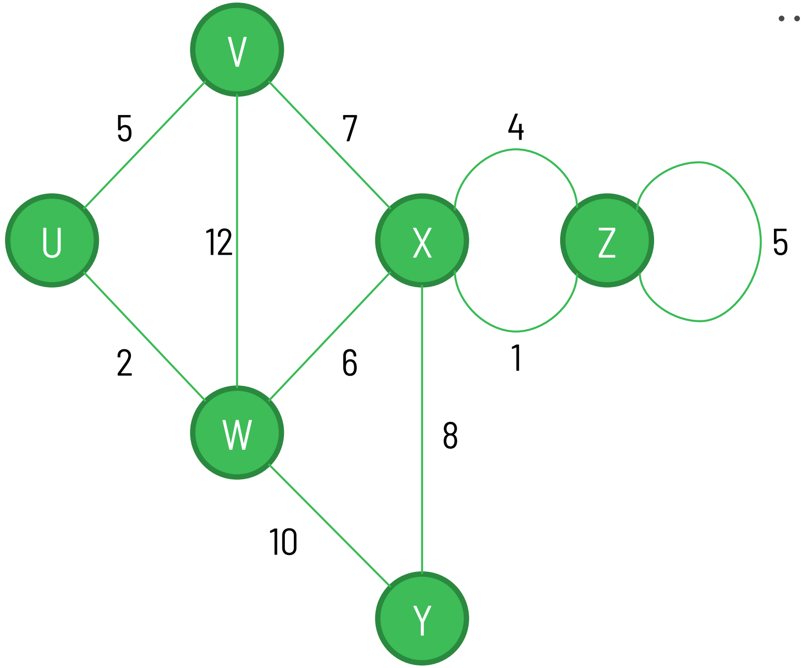
Definition and properties

Minimum Spanning Tree

The Minimum Spanning Tree (MST) of a graph G is:

- A spanning subgraph of G (contains all vertices from G)
- A tree (no cycles)
- Minimal weight-wise (sum of edge weights in minimum)

Caution: an MST does not guarantee the shortest path between two vertices.



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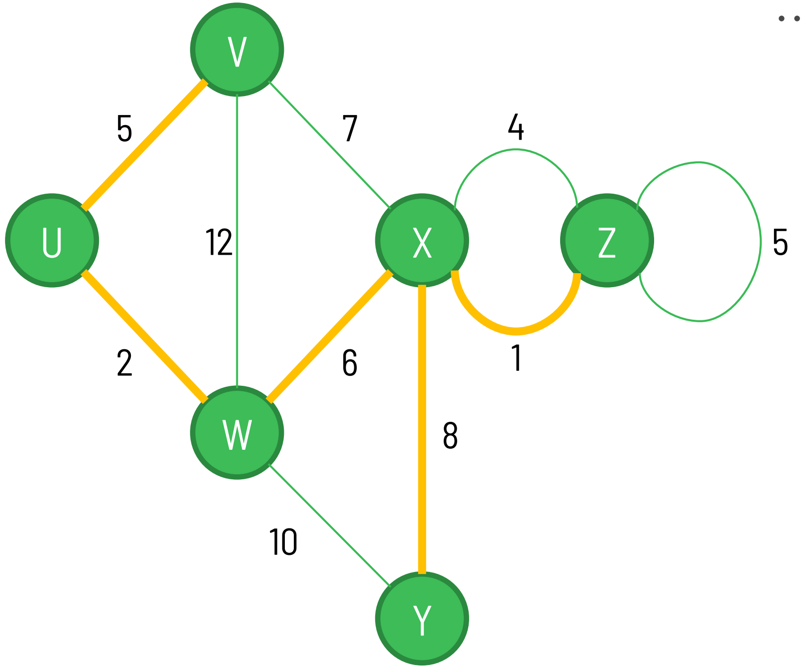
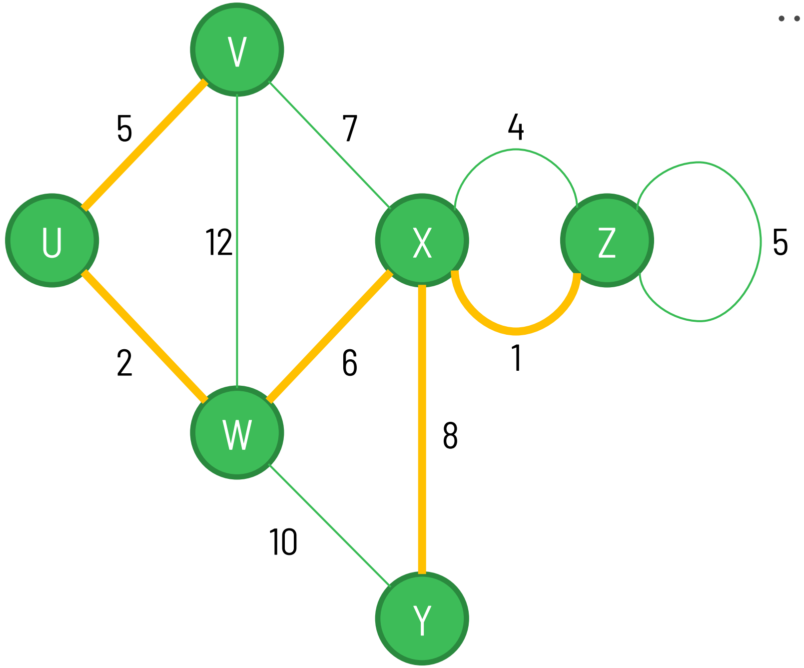


Cycle Property

Let T be a minimum spanning tree of a weighted graph G. Let e be an edge of G that is not in T. Let C be the cycle formed by adding e to T.

Claim:
For every edge $f \in C$, $\text{weight}(f) \leq \text{weight}(e)$

Proof by contradiction: Given the definitions above, assume that for some edge $f \in C$, $\text{weight}(f) > \text{weight}(e)$. Then, replacing f with e will produce a spanning tree T' such that its total weight is smaller than the weight of T. But that contradicts the definition of T as a MST of G.

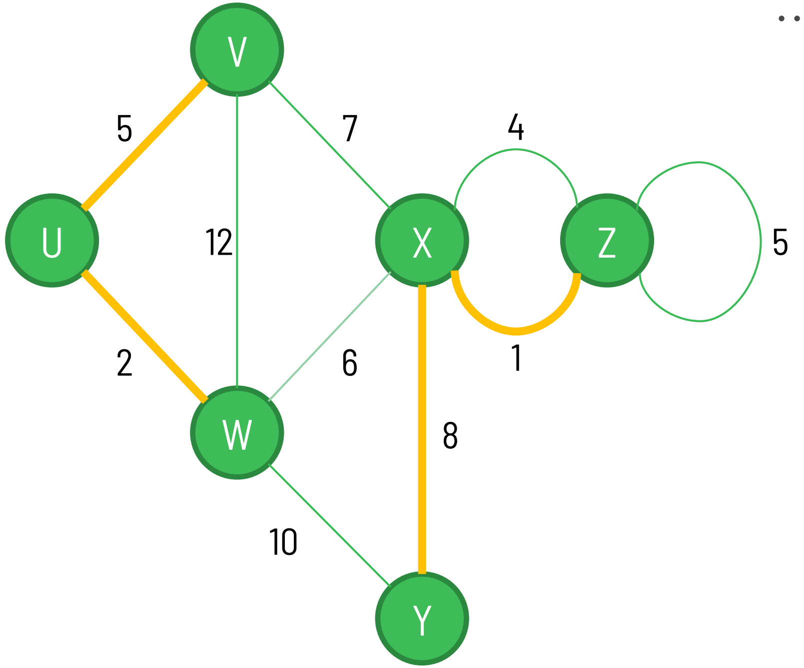


Partition (Cut) Property

Consider a partition of a graph G into subsets U and V. Let e be an edge of minimum weight across the partition (i.e., e has an endpoint in U and the other in V).

Claim: There is a minimum spanning tree of G that includes edge e.

Proof: Let T be an MST of G and let the definitions above be true. If T does not contain e, consider a cycle C formed by e with T and let f be an edge of C across the partition. By the cycle property, $\text{weight}(f) \leq \text{weight}(e)$. Thus, $\text{weight}(f) = \text{weight}(e)$, and we obtain another MST by replacing f with e.



Thinking about MSTs

- Given an undirected, weighted graph G, let T be an MST of G. Is T unique?
- Does an MST also give the shortest path between a pair of vertices?
- Does every weighted undirected graph have an MST?
- Can you find an MST of a weighted undirected graph if there are negative weight edges?
- Can you find an MST in a directed graph?
- How would we find an MST of a weighted undirected graph?

Minimum Viam Connectens

Do you have any questions?

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